DOCUMENT RESUME

ED 215 877 SE 037 068

AUTHOR Gilbert, Kathleen; Leitz, Steven
TITLE Problem Solving Styles Among Children Working in Small Groups on Mathematical Word Problems.
INSTITUTION Stanford Univ., Calif. Center for Educational Research at Stanford.
PUB DATE 82
NOTE 18p.; Paper presented at the Annual Meeting of the American Educational Research Association (New York, NY, March 18-23, 1982).

EDRS PRICE MF01/PC01 Plus Postage.
DESCRIPTORS *Cognitive Processes; Educational Research; Elementary Education; *Elementary School Mathematics; *Group Discussion; Group Dynamics; *Grouping (Instructional Purposes); Mathematical Applications; *Problem Solving; *Student Characteristics
IDENTIFIERS Heuristics; *Mathematics Education Research

ABSTRACT

This document focuses on four children in a small group, solving word problems aloud. Different aspects of how children in small groups approach problems were revealed; certain characteristics of their attempts to solve problems suggest a variety of questions for further research. These pupils were part of a project that involved six groups of students. The groups met once per week away from their regular classes. The pupils were instructed to take turns reading problems in each session and to write down the agreed solutions on a group answer sheet. The student discourse was recorded and coded. The discourse from the four children is presented as an initial step toward identifying what seem to be significant aspects of pupil communication, and the discussion is geared towards hypothesis-generating rather than theory-verifying. (MP)

****
* Reproductions supplied by EDRS are the best that can be made *
* from the original document. *
****

ED215877

1982 Annual Meeting
American Education Research Association
New York City

U.S. DEPARTMENT OF EDUCATION
NATIONAL INSTITUTE OF EDUCATION
EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

This document has been reproduced as received from the person or organization originating it.

☐ Minor changes have been made to improve reproduction quality.

• Points of view or opinions stated in this document do not necessarily represent official NIE position or policy.

Problem Solving Styles Among Children Working in Small Groups on Mathematical Word Problems

Kathleen Gilbert
Stanford University

Steven Leitz
Stanford University

Center for Educational Research at Stanford
Stanford University
Stanford, California 94305

Tel. 415/497-4891

"PERMISSION TO REPRODUCE THIS MATERIAL HAS BEEN GRANTED BY
KATHLEEN GILBERT
TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)."

SE 037 068

Problem Solving Styles Among Children Working in Small Groups on Mathematical Word Problems

Introduction

Recognizing the growing need for effective problem solving skills, the National Council of Teachers of Mathematics (1980) has recommended that problem solving be the focus in the research and teaching of school mathematics during the 1980s. There is evidence that students at all levels experience considerable difficulty in solving verbal problems. A 1979 survey, the National Assessment of Educational Progress, found that many nine, thirteen, and seventeen year-old students cannot choose the correct computational procedures to solve word problems, even though they may understand the particular operations required to perform the necessary computations. Teachers have commented that the teaching of story problems is one of the "thorniest" areas in the elementary mathematics curriculum (Blankenship and Lovitt, 1976). The reactions of students are perhaps best summarized by the words of a second grader who after reading a simple word problem said to her teacher, "I understand the words and the numbers well, just tell me whether to add or subtract" (Nesher, 1980).

An individual is confronted with a problem when enountering a question s/he cannot answer or a situation s/he is unable to resolve using the knowledge immediately available. The individual must think of a way to manipulate the information at his/her disposal to arrive at the goal, the solution to the problem. A problem is different than an exercise. To complete an exercise, one has an algorithm that when applied correctly will assuredly lead to the solution. First, one selects the applicable algorithm, then performs the necessary manipulation. Both steps are necessary, neither alone is sufficient. Similarly, problem solving can be described as having two general aspects: the process, or the heuristics that direct the search for the solution; and the product, or the actual solution. Both the processes and the product are essential components of

problem solving. With few exceptions, research in the area of mathematical word problem solving has concentrated on the product aspect of problem solving. Most studies are experimental in design and use scores on objective measures of achievement as the depedent variable. Little research has been conducted that focuses on the processes of problem solving. The need for this type of research was voiced as early as 1925 by Buswell and Judd who stated:

> The analysis of the mental processes of children in arithmetic reasoning presents a difficult, but by no means unsolvable problem. There is needed an experimental study to point out the steps of difficulty in the reasoning process and to determine, in concrete fashion, how children reason and what are the most obvious difficulties encountered."

Why is it insufficient to analyze the products of the children's thoughts, that is, the answers to the arithmetic problems in order to recommend effective teaching methods? Four fundamental reasons are apparent:

[1] Knowledge about students' problem solving processes may lead to recommendations for problem solving instruction that is more compatible to the students' actual problem solving styles. The observation that children in small groups spend a good deal of time evaluating their answers and explaining them to each other suggests that the typical pattern of teacher elicitation / student response / teacher evaluation may interrupt the children's natural problem solving processes.

[2] Merely considering the answers to word problems opens the possibility of reinforcing inefficient strategies the children may use to solve the problems.

[3] An incorrect answer standing alone communicates little to the teacher about the sources of errors. During the study described later in this paper, a word problem with fractions was tackled by a group of four fifth and sixth graders. "James ate 1/3 of a pizza. How much did he have left?" The answer given consensus by the group was 2/2, a seemingly nonsensical result. However, embedded in the discourse that led to the answer was a chain of nearly sound reasoning. The group reasonably determined that

the pizza originally had three pieces and James ate one, leaving two. The confusion arose in mistakenly transforming 1/3 to 1/2. This exemplified a surprisingly common tendency when these children worked with fractions, that is, the tendency to force 1/2 into the problem whenever fractions were indicated.

[4] Examining what children say while they solve word problems is likely to bring into question the commonly held assumption that right answers imply correct understanding and wrong answers imply wrong understanding. Erlwanger (1973) discusses Benny, a twelve year-old who said that although he usually didn't get the answers to decimal fraction problems on the first try, he could figure out what was wanted after having his answers checked against the answer key. Benny explained his work:

> Wait, I'll show you something. This is [has?] a key. If I ever get this one [i.e., 2 + .8] . . . actually, if I put 2 8/10, I get it wrong. Now down here, if I had this example [i.e., 2 + 8/10], and I put 1.0, I get it wrong. But really they're the same, not matter what the key says.

and later,

> I did 2 + .3, that will give me a decimal; that will be .5. If I did it in pictures [i.e., physical models] that will give me 2.3. If I did it in fractions like this [i.e., 2 + 3/10], that will give me 2 3/10. [Cited by Easley, 1977, p. 6].

Benny has an assortment of answers, one or two of which would be accepted as correct, and another that is clearly incorrect. He uses a trial-and-error method to find out the one that is acceptable according to the key even though he remains unconvinced.

Children in small groups solving word problems express a variety of styles and strategies often hidden in ordinary classrom activity. The analysis of their problem solving processis is inexorably bound to the analysis of the heuristics they use in solving word problems. The term *heuristic* has a lengthy history and a specialized meaning that goes beyond the original Greek definition of "serving to discover." Heuristics, classically defined by Polya (1945), include the mental operations typically useful in the process of solving problems. A set of specific questions to ask or steps to follow in

working on a problem would be characterized as heuristics. For example, Polya begins his list of heuristics with the three questions: What is the unknown? What are the data? What are the conditions? Heuristics are key strategies allowing the problem solver to proceed toward solution of the problem.

A serious impediment to the progress of research on the teaching and learning of word problem solving has occurred in the analysis of the heuristics children use. The examination and identification of heuristics children employ naturally and effectively is incomplete (Kantowski, 1977). A major stumbling block in describing effective heuristics is the inadequacy of techniques to determine the child's thought processes while solving word problems.

Researchers attempting to study problem solving processes have always had to contend with the difficulty of obtaining an external indicator of the inferred mental process. Many links in the mental process escape the investigator. We can see both the input and the output, but we cannot see the actual processing, the mental operations employed to arrive at a solution. Researchers can look at the teaching process, the textbook's presentation of problems, and the structural difficulty of the problems to obtain data relevant to the input to the student. Similarly, the output of the problem solving process can be identified by utilizing test scores, written records of an individual's solution path, or diagnostic interviews. But how does a researcher obtain information about the student's mental processing, the heuristics considered and employed, when the cognitive aspects of performance are not visible? This is the methodological problem that has plagued researchers concerned with the process of problem solving. How does one obtain an external indicator of the internal mental processing?

Although it is inevitable that a child's train of thought remain a "black box" phenomenon to some degree, there is much that an be learned by examining the outward manifestations of thinking. To reveal the qualitative aspect of the complex process of problem solving, the researcher must try to get the child to expose, as far as

possible, the actual process of thinking during the solution of a problem.

Two experimental methods used to investigate children's problem solving processes are the single student protocol and small group protocol. The single student protocol has dominated the research and is characterized by variations on the Piagetian clinical interview emphasizing overt thinking. A number of researchers, however, have pointed out serious limitations with this methodology for determining the heuristics children use (Bruner, Goodnow, and Austin, 1962; Webb, 1979; Zweng, 1979; Noddings, 1982). We believe it is profitable, then, to listen to students in an alternative setting. Supported by evidence that small groups tend to stimulate cognitive process (Sharan and Sharan, 1976; Johnson, 1981) and verbal expression (Slavin, 1979), Noddings (1981) has suggested that protocol analysis of small groups might provide samples of discourse that reflect the heuristics children use in mathematical problem solving.

The Children in Context

The focus of the analysis in this paper is four children in small groups solving arithmetic word problems aloud together. Their dialogue was audio-taped, transcribed, and coded during the San Jose/Stanford Teacher Corps Project. Each child participated in a different group of four eleven and twelve year-olds and the groups were recorded during four approximately twenty-five minute sessions. All of the children in this study belonged to the middle third of the local fifth and sixth grade according to their grade according to their grade equivalent scores on both the math and reading sections of the California Test of Basic Skills (CTBS).

Four specific children, Bert, Bette, Lora, and Rita, were selected for the discussion in this paper because each one reveals different aspects of how children in small groups approach math word problems. At the same time, the four share certain characteristics in their attempts to solve the problems and their discourse suggests a variety of questions for further research.

Six groups of children were assembled for this project. The four students within each group were relatively similar to one another with respect to their grade equivalent scores on CTBS, that is, between any two members within each group there was a spread of no more than 1.5 grade levels for either math or reading. Five of these groups consisted of two boys and two girls, and Lora's group, the sixth, contained four girls.

The groups met once per week away from their regular classes. They were instructed to take turns reading each of the eight problems in the session and to write down the agreed solutions on a group answer sheet. At the end of each session, they were given answer keys and were told to go back to any problems they had missed, trying to figure out how they could get a correct answer.

The arithmetic word problems given to the students were variations on eight different general forms of problems taken from a computer-based problem solving curriculum, *Problem Solving* (Noddings, 1978). The problems across sessions were generically equivalent but appeared different. This design permitted more description of the changes in the facility with which the children solved the problems throughout the sessions.

Listening to the Children's Talk

The coding of the discourse units took place on two distinct levels: sociolinguistic and mathematical. The sociolinguistic was concerned with the verbal structuring of the discourse, that is, initiating, responding, or following up main ideas, the general domain of each discourse unit (ritual, feeling, or math), the grammatical form of the unit, and the specific communicative function of the unit. A discourse unit (or speech act) was defined to be an utterance or part of an utterance which can stand by itself as a complete thought. The portions of the discourse coded in the mathematics domain are of particular concern in this paper. The mathematical analysis coded the problem solving processes as suggesting, seeking, or using an explanaiton, an algorithm, a

numerical answer, or a heuristic. These primary categories were further divided into sub-categories.

This detailed, arduous coding procedure has provided a rich source of data on whih computerized searches and counts of various discourse unit combinations can be performed. A seemingly limitless array of questions can be investigated with respect to this data. The coded discourse lends itself to:

- identification and analysis of differences between the problem solving styles of girls and boys;
- descriptions of the kinds of problem solving processes revealed in talk directed toward oneself compared to group-directed talk;
- identification of general problem solving styles indicated, for example, by reflection-on-process versus reflection-on-answer, visualization, and degree of emphasis on responding to the content rather than the form of the problem;
- discussion of strategies that seem to determine effective group participation, that is, how some group members exert more control than others over both the topics and group consensus;
- analysis of the types of explanations children offer and accept in order to verify their answers.

This paper, however, considers examples of discourse from four children as an initial step toward identifying what seem to be significant aspects of their communication in small groups while solving math word problems. The following discussion is hypothesis-generating rather than theory-verifying. As the hypotheses are discovered and clearly formulated, systematic counts and statistical analyses can be performed in an attempt to evaluate the generalizability of these examples.

Bert

Bert's discourse is characterized by frequent attempts to be the first one in the group to provide a correct answer. He announced after Problem 1 in the first session, "I'm the first one so I'm the smartest here." He had indeed been the first to produce the right answer, but it was questioned and then immediately challenged by other members of the group. Gradually, however, the students rather independently corrected their addition errors and reached consensus on the correct answer. Typical of almost all the groups' dialogue, they did not acknowledge nor offer praise for correct answers or clever solutions.

Once again, in Problem 3, Bert quickly produced an answer. The other children appeared baffled by the problem:

> Here is a number: 4,963. Use these 4 digits to form the number with the least value.

Bert's first answer, promptly after the reading of the problem, was 9,653, actually the right answer in reverse order. Only fourteen speech acts later he suggested the correct result, 3,569. That failed to capture the attention of the group and when Ann arrived at 1,101,852 dollars as a possible answer, Bert said, "Kill yourself." Then six acts later he explained. "What I think is . . . is you start by the . . . start . . . use these four numbers and you start by the less one, the smaller one and go up."

The interplay of Bert's social relationship with the group and his invented, somewhat abstract rule, to explain his answer created a mild tension between Bert and the other children. Ann announced, "I don't know what he's talkin' about." Yet Bert's suggested algorithm seemed to take root in the mind of another group member and 102 speech acts after Bert's explanation, his answer was reluctantly accepted. Ann asked, "Is that right?" and then acquiesed with, "Okay, just write it."

This example simultaneously points out difficulties and positive effects while small groups interact and solve math word problems. Bert's need to be first with the answers,

often in competition with Ann, is likely to have stifled the expression of some problem solving strategies. Further study of similar small groups might well examine the effects of status equalization techniques in assembling and training the groups (Cohen 1980) on the quality of the discourse. On the other hand, in spite of the strains within the group, the children were drawn to the task of solving the math word problems. Bert, as well as the other children, revealed a rich variety of problem solving strategies, and during the last session he told the researcher that he learned to "do math better by us helping each other." The hypothesis that small groups do help children "do math better" is appealing and suggests the need for well-designed research to clarify in what ways small groups contribute effectively to children's problem solving skills.

Bette

One strategy children occasionally applied to the word problems was to make the situation in the problem more concrete, more directly related to their own experiences. Several modes of concretizing the problems appeared in this study, although the general strategy occurred infrequently. Bette, for example, verified an answer to a problem by checking it with her own experience. After the children had agreed on $1.29 as the answer, Bette announced, "These days you buy things for that much . . . probably around a dollar something."

Similarly, as the group tried to reach consensus on the problem:

> Tom is 17 years old. Tom is 6 years younger than Nancy. How old is Nancy?,

a student in the group lightheartedly brought the characters of the problem to life.

> Bette: . . . years, older.
>
> Ezra: Younger.
>
> Dan: Younger, man. It's not old . . .
>
> ——: [laughter] She's 20 . . . She's his girlfriend.

Further discussion resulted in the correct answer, 23, but the sideline inference and estimated answer suggest that the students naturally tried, at times, to incorporate textbook problems into their personal terms.

During another problem about how many stamps someone saved in a week, Bette and Dan debated the number of days in a week. Bette finally became convinced that a week contains seven days rather than five.

> Bette: I know, Monday, Tuesday, Wednesday, Thursday, and Friday. That's 5 days: a week .....
>
> Dan: It ain't no week. It's school days, man.
>
> Ezra: Those are school days. A week is with Saturday and Sunday ....
>
> Bette: Ohh ... hum, okay.

The contrast between school days and Saturday and Sunday demonstrates another, if somewhat unsophisticated, attempt to make the conditions of the problem concrete.

Bette, like the other children, did not employ a large variety of strategies to concretize the problems. Nevertheless, visualization of the problem situation was revealed by some children as a further example of this general tendency to reduce the level of abstraction within the problems. The pizza problem elicited attempts to represent the question in drawings of circles. (difficulties arose, however, whenever the children tried to divide their circles into fractional parts.) The pizza problem in a form such as:

> Sue ate 1/5 of a pizza. How much did she have left?,

consistently resulted in the children's question, "But how big is the pizza?" During one discussion, Sam concludes that "it would be a small if she was by herself." This provides further evidence that the children were trying to concretize the problems.

Bette's most frequent strategy for working the problems concretely rather than abstractly was to resort to methods of counting:

- "... 26, 27, 28, 29, 30, ... 30. It's 30."

- "4 digits is 4. 1, 2, 3, 4."
- "18. 2, 2, 4, 6, 8, 10 . . ." * finishes counting silently *
- "Okay, 5, 6, 7, 8, 9, 10, 11, 12, 14. She is older than him, so count 5, and then 9. Okay?"

Lora

Lora was a member of the group of four girls. This group demonstrated a strikingly different style than the other groups composed of two girls and two boys. Another interesting research question considers the generalization that single sex groups operate differently than mixed groups, and that groups of girls generate different styles of problem solving behavior than all boy groups. Whether or not Lora's particular role would have changed in a mixed boy/girl group is also open to question.

Lora quickly assumed the role of directing the group's procedures as the following comments indicate:

- " 'k, read it, Nanette."
- "You know it, Lynn?"
- "Do you all agree on that anwers?"
- "Okay, read number 4, Darby."
- "Everybody agree on 22?"

Atypical of almost all the children's discourse, Lora even offered, "You're smart," as praise to another child for explaining an answer.

A concern for procedure and form also dominated Lora's problem solving style. In her mind, understanding the problem appeared to mean having a procedure to get the right answer. When the researcher asked Lora if there were any way to be sure the

answers were right, Lora answerd, "Yeah, give us the answers and then check 'em." The researcher asked if there were any other way. Lora said, "No."

Lora's method of solving the word problems largely depended on trial-and-error, reminiscent of Benny's rules and conceptions of mathematical thinking (see Introduction). During a review of the answer key, the girls were describing how they solved a problem.

> Nanette: We had to figure out an answer.
>
> Lora: We had to figure out the answer out of subtracting or adding or . . . we picked adding but . . .

Then when asked why it made sense to subtract,

> Lora: So you can get the answer.

Earlier Lora had explained:

> First you times it, ya plus it, then ya . . . take away it and then you . . . see which one you want.

The potential of the small group to dissuade children from inefficient or inappropriate problem solving strategies emerged gradually as her peers gently rebuked Lora's "add-subtract-or-multiply" approach.

> Lora: So it's either 72 or . . . 94 or . . .
>
> Darby: Lora!

An important question for empirical research, then, is if small group interaction can reduce the frequency of inappropriate problem solving strategies over time.

Rita

The possibility that some children will remain nearly silent during the small group dialogue is exemplified by Rita. A record of the number of times she responded during the eight problems of her first session is shown below.

Session 1

Prblm	#Rita's Responses	Total#Group Resp.	#Rita's Resp./Total
1	0	18	0
2	2	14	.14
3	1	29	.03
4	4	16	.25
5	1	17	.06
6	3	31	.10
7	3	22	.14
8	0	7	0
Total	14	154	.09

The actual number of her responses increased by the last session as did her percentage of the group's total. Nevertheless, her participation continued to remain very low.

Session 4

Prblm	#Rita's Responses	Total#Group Resp.	#Rita's Resp./Total
1	2	12	.17
2	0	6	0
3	2	12	.17
4	5	35	.14
5	5	57	.09
6	1	23	.04
7	3	13	.02
8	4	15	.27
Toal	22	173	.13

This clearly presents a problem if one assumes that a primary benefit of small groups is active individual participation. However, further research must be carried out in order to determine whether or not gains in problem solving skills can be made by "silent" participation.

Future Directions

Two aspects of the study discussed here will guide our future research plans. First, in over 90% of the problems completed by the small groups, an answer was offered within three acts of the reading of the problem. In other words, the problems were more like exercises than like "problems." For many of the children, the solution or at least the plan for solving the "problem" was so apparent that no "problem" existed. The vast majority of the dialogue then would center around verifying the proposed answer. In such situations, the opportunity and stimulation for generating problem solving heuristics or for "reflexive talk" (Noddings, 1981) is severely limited.

A second aspect may be characteristic of small groups in general. Although the researcher may get richer data from some of the children through their "reflexive talk", other children in the small groups reveal almost no information about their thinking processes. In all of the groups examined, there was, consistently, one individual who was credited with less than 10% of the total acts. In one particular group, one of the girls uttered only fifteen phrases throughout the entire four sessions. Further, in every group examined, one member gave over half of the answers to the problems while another member offered fewer than two answers. These figures illustate that the small group protocol may yield very rich data on some individuals and very little data on others.

Nevertheless, it appears that the small group protocol method provides a viable alternative for revealing the qualitative aspects of the complex process of problem solving. Our future research will concentrate on optimizing the benefits of the technique. Special attention needs to be given to variations in the composition of the groups by sex, by math and verbal ability, by status and sociability, to the pretraining of the group, and to the difficulty of the problems. As we learn more about how these variables influence small group discourse, we will begin to realize the full potential of the small group protocol as a means of exposing the mathematical problem solving heuristics of children.

References

Blankenship, C. S. and Lovitt, T. C. Story problems: Merely confusing or downright befuddling? *Journal for Research in Mathematics Education*, 1976, *7*, 290-298.

Bruner, J., Goodnow, J., and Austin, G. *A study of thinking*. New York: Science Editions, Inc., 1962.

Buswell, G. T. and Judd, C. H. *Summary of educational investigations relating to arithmetic*. Supplementary Educational Monograph, No. 27. Chicago: University of Chicago Press, 1925.

Cohen, E. G. *Status Equalization Project: Changing Expectations in the Integrated Classroom*. Unpublished final report, Stanford University, 1980.

Erlwanger, S. H. Benny's conception of rules and answers in IPI mathematics. *Journal of Children's Mathematical Behavior*, 1973, *1(2)*,7-26.

Johnson, D. W. Student-student interaction: The neglected variable in education. *Educational Researcher*, 1981, *10*, 5-10.

Kantowski, M. G. Processes involved in mathematical problem solving. *Journal for Research in Mathematics Education*, 1977, *8*, 163-180.

National Assessment of Eductional Processes. (NAEP) (1979), Second assessment of mathematics: Mathematical applications. Report No. 09-MA-03, Educational Commission of the States. Denver, Colorado.

National Council of Teachers of Mathematics (NCTM). Agenda for action, recommendations for school mathematics of the 1980s. Reston, Virginia: NCTM, 1980.

Nesher, P. The stereotyped nature of school word problems. *For the Learning of Mathematics*, 1980, *1(1)*, 41-48.

Noddings, N. et al. How children approach math word problems. Paper presented at annual conference of American Eductional Research Association, Los Angeles, 1981.

Noddings, N. *Problem Solving. Grades 3-6.* Palo Alto, CA: Comuter Curriculum Corporation, 1978.

Noddings, N. The use of small group protocols in analysis of children's arithmetical problem solving. Paper presented at annual conference of American Educational Research Association, New York City, 1982.

Polya, G. *How to solve it.* Princeton: Princeton University Press, 1945.

Sharan, S. and Sharan, Y. *Small-group teaching.* Englewood Cliffs, N.J.: Educational Technology Publications, 1976.

Slavin, R. E. Cooperative learning in teams: State of the art. Invited address, American Psychological Assoc., New York, 1979.

Webb, N. L. *Learning in individual and small group settings.* Unpublished doctoral disseration, Stanford University, 1977.

Zweng, M. *Children's strategies of solving verbal problems,* NIE grant No. G-78-0094, August 1, 1979.